Rudolf Carnap's Approach to the Problem of Induction

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Induction

Carnap's Motivation for an Inductive Logic

Hume's problem lurking in the background:

> *"It seems to me that the view of almost all writers on induction in the past and including the great majority of contemporary writers, contains one basic mistake. They regard inductive reasoning as an inference [...]. From this point of view the result of any particular inductive reasoning is the acceptance of a new proposition[. ...]*
>
> *This seems to me wrong. On the basis of this view it would be impossible to refute Hume's dictum that there are no rational reasons for induction. [...]*
>
> *I would think instead that inductive reasoning about a proposition should lead, not to acceptance or rejection, but to the assignment of a number to the proposition, viz., its [degree of confirmation]. (cf. 1966, p.317f)*

Carnap's Suggestion

Traditional Approach:

Acceptance of hypothesis S_{n+1} on the basis of evidence $S_1, \ldots, S_n$.

S_1 Swan$_1$ is white.

S_2 Swan$_2$ is white.

$\vdots$

S_n Swan$_n$ is white.

S_{n+1} Swan$_{n+1}$ is white.

$\Rightarrow$

Carnap's Approach:

Determination of the logical probability of hypothesis S_{n+1} given evidence $S_1, \ldots, S_n$.

$$\mathfrak{c}(S_{n+1}|S_1, \ldots, S_n) = r$$

A Problem of Logical Empiricism

Historically, we distinguish between:

- **Rationalists:** reliance on *synthetic a priori* statements; based on a deductive methodology only
- **(Logical) Empiricists:** committing *synthetic a priori* statements to the flames; includes also an inductive methodology

When discussing the "presuppositions of induction" (cf. Carnap 1950/1962, section F, §41), Carnap emphasised that this reliance on an inductive methodology was traditionally seen as, ultimately, countering empiricism.

The reason is, that the traditional justification of the inductive methodology hinges on the principle of the uniformity of nature:

> *"The statement of the probability of uniformity is [sometimes regarded] as a synthetic, factual statement [. . .]. But it cannot be confirmed empirically because such a procedure would use the method of induction which in turn presupposes the statement. Thus, they say, at this point empiricism must be sacrificed." (1950/1962, p.180)*

A Problem of Logical Empiricism

Here is an explicit version of this argument:

- E1 The justification of the inductive methodology hinges on the assumption about the *uniformity of nature*.
- E2 The assumption about the uniformity of nature is *synthetic* because it cannot be justified deductively (in which case it would be analytic, but deduction is too weak for justifying this assumption).
- E3 The assumption about the uniformity of nature is *a priori* because it cannot be justified inductively (in which case it would be a posteriori, but this would be circular, as we see by the help of premiss 1).
- E4 Hence, in order to justify the inductive methodology, one needs to make an assumption that is synthetic a priori, which counters logical empiricism.

We see Hume's dilemma operating here: the lack of strength of deduction for justifying induction is part of premiss E2 and the circularity of an inductive justification is part of premiss E3.

A Problem of Logical Empiricism

Ad premiss E1: Why was and is the assumption about the uniformity of nature so central for the inductive methodology?

- At the time when Carnap started to work on this topic, one important reason was the arise and establishment of the frequentist account of the inductive methodology and its conception of probability:
- Simplified speaking, the inductive methodology consists in straightforwardly extrapolating the relative frequency of an observed part of a series of events to the (next) unobserved part of that series of events.
- If nature—i.e. here the series of events in question—is uniform, then this extrapolation is adequate because we simply transform a pattern from the observed to the unobserved part and uniformity guarantees that patterns of the observed part are, indeed, also patterns of the unobserved part
- So, *frequentism* can be justified or even hinges on *uniformity* (cf. premiss E1 above), but for the same reason we cannot use frequentism to justify the uniformity assumption (cf. premiss E3).

Carnap's Account to the Problem of Induction

Carnap aimed at salvaging empiricism by putting it on logical ground (analogously Carnap's teacher Frege aimed at salvaging mathematics by putting it on logical ground):

1. He argued that in science and philosophy of science two conceptions of probability are in place, the logical and the frequentist conception (cf. his 1945).
2. He argued that very often the logical component of the inductive methodology is what is relevant for science and philosophy of science
3. Particularly, he claimed that it is also the logical conception that can be employed in order to argue for *a* uniformity assumption in order to tackle the problem of induction.

This approach counters premiss E2 of the argument above and makes the respective uniformity assumption an *analytic a priori* statement.

So, according to Carnap, we do not have to "sacrifice empiricism".

Carnap's Account to the Problem of Induction

Here is what Carnap took to be the relevant uniformity assumption:

> *Justification of Uniformity:*
> *"On the basis of the available evidence it is very probable that the degree of uniformity of the world is high." (Carnap 1950/1962, p.180)*

So, the relevant uniformity assumption is a logical probabilistic statement about the uniformity hypothesis (h_u: the world is uniform) given the available evidence (e: about the framework—cf. below).

Here is the argument for justifying induction:

- J1 The justification of the inductive methodology (h_i: it is successful) hinges on the assumption about the *uniformity of nature* (same premiss as E1); let us, for reasons of simplicity, express this as a deductive/conceptual/analytic relation: $\vdash h_u \rightarrow h_i$
- J2 *Justification of Uniformity*: On the basis of the available evidence it is very probable that the degree of uniformity of the world is high, i.e.: $Pr(h_u|e)$ is high
- J3 Hence, on the basis of the available evidence it is also very probable that the inductive methodology is successful, i.e.: $Pr(h_i|e)$ is high

Carnap's Account to the Problem of Induction

Since J1 is conceptual, we take it for granted.

So, the justification of the inductive methodology ($Pr(h_i|e)$ is high) hinges on J2. How can we argue for this?

When publishing his (1950/1962), he announced the discussion of this topic for a second volume:

> *"The second volume will also [...] formulate and discuss the problem of the assumption of the uniformity of the world and its alleged necessity for the validity of inductive reasoning in a more exact way than in the present volume" (cf. preface, p.ix; similarly at p.495).*

However, it seems that he got carried away by technical details and never took up this justification again.

This outsourcing of argumentation was also noted by Nagel (1979).

Carnap's Account to the Problem of Induction

In principle, Carnap's idea was to analytically extract a justification of uniformity (h_u) from framework properties.

Although he never worked this out out in detail, there seems to be a way to do so: In the next section, we will present a reconstruction of an argument in favour of J2.

In the subsequent section, we also want to argue that uniformity considerations played an important role in more pragmatic successors of Carnap's programme.

However, let us see first in which sense Carnap provides a solution to the problem of induction.

Two Problems of Induction

Carnap's solution in a nutshell:

- The probable success of induction hinges on uniformity.
- The probable truth of uniformity is given via framework conditions.
- Hence, the probable success of induction is given via framework conditions.

So, we have two problems to solve here:

- Internal Problem of Induction: what are the logical probabilistic properties of a framework (e.g.: is h_u logically probable in the framework)?
  $\Rightarrow$ cf. next section;
- External Problem of Induction: which framework to choose?
  $\Rightarrow$ cf. subsequent section;

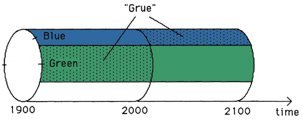
How Carnap's Account Bypasses Hume and Goodman

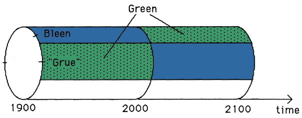
Ad Hume:

By avoiding acceptance:

> *[The difference between the acceptance of a hypothesis and the assignment of a degree of confirmation] may perhaps appear slight; in fact, however, it is essential. If, in accordance with the customary view, we accept the prediction, then Hume is certainly right in protesting that we have no rational reason for doing so, since, as everybody will agree, it is still possible that [our prediction is wrong]. (cf. 1966, p.318)*

Ad Goodman (blue/green vs. grue/bleen):





By stressing the framework-relativity: blue/green vs. grue/bleen framework

Probability

General Overview of Carnap's Work

- 1945: his first published systematical approach on probability; distinction of *probability*$_1$ (*degree of confirmation*/*logical probability*; e.g. Keynes (1921), Jeffreys (1939/2003)) vs. *probability*$_2$ (*relative frequency*; e.g. von Mises (1928/1957), Reichenbach (1935/1949))
- 1945: Outline of his programme of *logical probability*
- 1950/1962: worked out details of the programme within a full monograph devoted to this topic (investigated conditions for different families of confirmation functions ($\mathfrak{c}$ functions)
- 1952: discovery/definition of a continuum of inductive methods
- 1963: important parts of the discussion of his early system of an inductive logic
- 1966, 1957: re-stating of the aim of an inductive logic at several occasions as that of defining the concept of *probability*$_1$ or *confirmation* in "a purely logical way"
- 1971, 1980: technical development of further systems of inductive logic with which he seeked to overcome severe restrictions of the defined continuum of inductive methods

The Idea of an Inductive Logic

The programme of an inductive logic sees an important continuity with respect to deductive and inductive reasoning.

Traditionally: A is a logical consequence of B iff for all interpretations $\mathfrak{I}$ it holds that if B is true at $\mathfrak{I}$, so is A.

A machinery of deductive and inductive logic on the basis of propositional (modal) logic semantics:

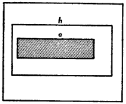
- *possible world*: a maximally consistent set of formulæ of some propositional (modal) language $\mathfrak{L}$
- *state description*: a set of formulæ of $\mathfrak{L}$ such that if φ is any atomic formula in $\mathfrak{L}$, a state-description for $\mathfrak{L}$ must either affirm or deny φ
- $\mathfrak{Z}$: let $\mathfrak{Z}$ of $\mathfrak{L}$ be the set of all state descriptions of $\mathfrak{L}$.
- *range* of a formula φ in $\mathfrak{L}$: the class of those elements of $\mathfrak{Z}$ in $\mathfrak{L}$ in which φ holds

The Idea of an Inductive Logic

It is often said that in a deductive inference, the "content" of the conclusion is already contained in the "content" of the premiss.

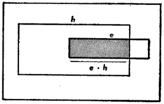
In our setup, we can express this explicitly via the range: If we take the "content" of a formula as the range-conditions (intension) and the range of a formula as its extension.

In accordance with the systematic intension-extension inversion of the direction of implication we can state that A is a logical consequence of B iff the range of B is contained in/a subset of the range of A (Carnap 1950/1962, cf. p.297):



Deductive Logic

'*e* L-implies *h*' means that the range of *e* is entirely contained in that of *h*.



Inductive Logic

'c(h,e) = 3/4' means that three-fourths of the range of *e* is contained in that of *h*.

The Idea of an Inductive Logic

Let us illustrate this by the help of an example. Take, e.g., the two formulæ p_1 and $p_1 \& p_2$. It is clear that p_1 is a logical consequence of $p_1 \& p_2$ but not vice versa. We can see this by the help of a simple truth table:

i	p_2	p_1	$p_1 \& p_2$
1	0	0	0
2	0	1	0
3	1	0	0
4	1	1	1

Whenever $p_1 \& p_2$ is true, so is p_1 but not vice versa. The "range" of $p_1 \& p_2$ (line 4) is contained in the "range" of p_1 (lines 2 and 4).

The Idea of an Inductive Logic

So much for the direction from $p_1 \& p_2$ to p_1.

But what about the other direction, from p_1 to $p_1 \& p_2$?

Here is where inductive logic seeks to pop in. The idea is to fine-grain the range-inclusion claim from included/not included to stating a degree of inclusion.

How much of A is included in B:

$$Pr(A|B) = \frac{|\{s_i : \mathfrak{I}_i(A) = 1 = \mathfrak{I}_i(B)\}|}{|\{s_i : \mathfrak{I}_i(B) = 1\}|}$$

Example:

$$Pr(p_1 \& p_2 | p_1) = \frac{|\{s_i : \mathfrak{I}_i(p_1 \& p_2) = 1 = \mathfrak{I}_i(p_1)\}|}{|\{s_i : \mathfrak{I}_i(p_1) = 1\}|} = 0.5$$

The Idea of an Inductive Logic

Now, of course, from an inductive standpoint, propositional logic is boring.

Carnap focussed on monadic languages (why monadic? because its metric is most simple and clear).

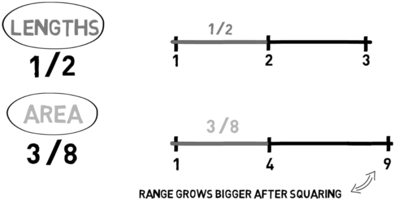
E.g.: Given one predicate B and three individuals a, b, c we get 8 state descriptions:

$$\begin{array}{cccc}
1 & \neg B(a) & \neg B(b) & \neg B(c) \\
2 & \neg B(a) & \neg B(b) & B(c) \\
3 & \neg B(a) & B(b) & \neg B(c) \\
4 & \neg B(a) & B(b) & B(c) \\
5 & B(a) & \neg B(b) & \neg B(c) \\
6 & B(a) & \neg B(b) & B(c) \\
7 & B(a) & B(b) & \neg B(c) \\
8 & B(a) & B(b) & B(c)
\end{array}$$

The Idea of an Inductive Logic

An important probabilistic principle in the history of probability is the principle of indifference: in the absence of any relevant evidence, probabilities are equally distributed among all possible outcomes under consideration.

Problem: inconsistency in naïve application (e.g. Bertrand's Paradox):



Carnap suggested to employ only linguistically sophisticated version.

The Idea of an Inductive Logic

Carnap: probabilistic indifference among structure descriptions (i.e. sets of state descriptions that are invariant w.r.t. the permutation of individuals).

E.g.: We can cluster the state descriptions to 4 structure descriptions:

1	$B(a)$	$B(b)$	$B(c)$
2	$\neg B(a)$	$B(b)$	$B(c)$
2	$B(a)$	$\neg B(b)$	$B(c)$
2	$B(a)$	$B(b)$	$\neg B(c)$
3	$\neg B(a)$	$\neg B(b)$	$B(c)$
3	$\neg B(a)$	$B(b)$	$\neg B(c)$
3	$B(a)$	$\neg B(b)$	$\neg B(c)$
4	$\neg B(a)$	$\neg B(b)$	$\neg B(c)$

A Solution to the Uniformity Problem

In fact, we can analytically extract a justification for uniformity (h_u) from the framework:

Structure descriptions that exhibit more uniformity are instantiated by a smaller number of state descriptions, so, "more uniform" state descriptions receive a relatively greater initial probability (cf. Leitgeb and André W. Carus 2020, Appendix C):

1	$B(a)$	$B(b)$	$B(c)$
2	$\neg B(a)$	$B(b)$	$B(c)$
2	$B(a)$	$\neg B(b)$	$B(c)$
2	$B(a)$	$B(b)$	$\neg B(c)$
3	$\neg B(a)$	$\neg B(b)$	$B(c)$
3	$\neg B(a)$	$B(b)$	$\neg B(c)$
3	$B(a)$	$\neg B(b)$	$\neg B(c)$
4	$\neg B(a)$	$\neg B(b)$	$\neg B(c)$

So, $Pr(h_u | \text{this framework})$ is high.

internal solution to the problem of induction ✓

Problem with the Programme of an Inductive Logic

Now, there are several problems with the programme of an inductive logic.

Two central problems are the "logical" determination of ...

- the probability of a genuinely universal statement

  (in many systems it holds that $Pr(\forall x B(x)) = 0$)

- the probability of (imperfect) analogies

  (in many systems $Pr(B(c)|B(a), B(b))$ or $Pr(B(a_{n+1})|B(a_1), \ldots B(a_{n-1}), B(a_n))$ are inadequate)

This led Carnap and others to introduce several free parameters: λ, μ, γ.

Which make the choice less logically determined (and looking more like a degenerative research programme).

Pragmatic Elements

The Role regarding Bayesianism

Very briefly the idea regarding Bayesianism:

- Frequentists: only observed and extrapolated frequencies are relevant; everything is dynamically determined by the observed frequencies; no relevance of priors; (only probability$_2$)
- Bayesians: priors are relevant;
  - subjective version: no restrictions on priors
  - objective version: objective restrictions on priors (somehow related to frequencies; principle principle etc.)
  - Carnap: logical determination of priors (framework properties) $\Rightarrow$ logical Bayesians (in the same spirit: complexity thinking of Solomonoff et al.); so, also probability$_1$

Perhaps we should distinguish à la Carnap between probability$_1$, probability$_2$, probability$_3$, ... (i.e. there are different explicanda asking for different explicata)

The Role regarding Conceptual Spaces

In his final publication, Carnap brought in geometrical thinking.

Later on, Gärdenfors (2000) worked on *conceptual spaces* in a similar line (cf. Sznajder 2016), though Gärdenfors mentions Carnap only very briefly.

It is interesting to see that Carnap seem to have become way more pragmatic than logical (more free parameters ... regarding the underlying metrics etc.).

At the same time, Gärdenfors' work seem to allow for making a step back more towards a "logical" approach (convexity as a very general criterion for clustering concepts).

One might even link this back to the problem of induction and the question of uniformity (Gärdenfors launches a convexity-argument vs. Goodman).

The External Problem of Induction

For Carnap, the choice of a framework is a practical question:

> *"I regard the external questions themselves, like the examples just mentioned, as practical questions." (cf. Carnap 1963, p.982)*

There are passages where he suggests to apply *expected utility* reasoning to practical questions, also the choice of a framework.

His general idea: logical probabilities (define what's expected) and utilities make up for a basis for decisions.

Problem: If also the choice of a framework is such a kind of decision, one can launch a regress argument (cf. Steinberger 2016; and a more general discussion in A. W. Carus 2017).

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